HOW WELL THE ANALYSIS OF AN INS/GPS INTEGRATION ALGORITHM CAN BE DIVIDED INTO TWO PROCESSES.

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ABSTRACT

This work deals with approximated performance evaluation for airborne navigation systems composed of GPS and low cost inertial sensors. Usually, performance evaluation requires to design a multivariable Kalman filter and to perform intensive numerical studies. This work follows [1] and proposes to consider an INS/GPS integration algorithm as a fusion of two processes. The first one is called filtering and deals with the sensor error model. It is aimed to calculate the high order position derivatives (velocity, acceleration, jerk). The second process is named separation and its goal is to calculate, from filtering results, all navigation and sensor errors. The most important feature of the above partition is the ability to analyze independently sensor error models and flight trajectories.

This paper analyzes the accuracy achieved by the calculations based on filtering-separation partition. First the underlying assumptions are precisely defined. The errors introduced by those assumptions, for different trajectories and sensor models, are analyzed. Algorithms for filtering and separation are discussed. Finally, leveling errors calculated by the combined filtering-separation algorithms are shown to be very close to the ones calculated by the standard, complete Kalman filter.

1 INTRODUCTION

An integrated Global Positioning System (GPS) receiver with an Inertial Navigation System (INS) is widely used as a precise, high bandwidth, robust, position, velocity, and attitude data source. See [6], [7] for a general review and discussion about benefits of INS-GPS systems. A standard technique to integrate INS with GPS is to design an appropriate Kalman filter. The integrated system is highly coupled, it is time, trajectory, and sensor error model dependent. Therefore, mainly because of the need to carry out intensive numerical studies, the task of designing a Kalman filter is time consuming. Consequently, a tool to evaluate approximately, but easily and fast, the system performance is highly desirable. This paper makes an attempt to create a general framework, in which the influence of the sensor error model and the flight trajectory are analyzed easily and independently.

The idea is to divide the entire estimation algorithm into two processes: filtering and separation. Filtering is aimed to estimate quantities like position, velocity, acceleration, and jerk. They are described by a time-invariant system. This system takes into account the statistical sensor error model. The second process, assumes as known the results of filtering, mainly acceleration and jerk, and calculates all unknown navigation and sensor errors, for example attitude, gyro drifts, or accelerometer biases. This process is called separation and it deals with the flight trajectory.

The goal of this work is to quantify how well, for different trajectories and sensor error models, this partitioning works. The paper is organized as follows. First the standard strapdown navigation error model is presented. Some model simplifications are proposed and justified for medium (tactical) or low (automotive) quality of inertial sensors. The next proposition is to consider the flight trajectory as consists of level straight legs, with short turns connecting consecutive segments. Our performance criterion is attitude estimation accuracy. We conclude that for a general trajectory, the partition into equivalent straight segments is related to properly identified time constants. Those time constants are associated with acceleration and can be calculated by filtering problem analysis. Finally the filtering and separation algorithm are discusses, their combined results are successfully compared with the standard Kalman filter calculations.

2. MATHEMATICAL BACKGROUND

The standard error model for a strapdown system is as follows:

$$\dot{\underline{r}} = \underline{w}_r \times \underline{r} + \underline{v} \tag{1}$$

$$\dot{\underline{v}} = \underline{w}_v \times \underline{v} + \underline{f} \times \underline{\psi} + \underline{\delta g} + C_{BL}\underline{B} \tag{2}$$

$$\dot{\underline{\psi}} = \underline{w}_\psi \times \underline{\psi} - C_{BL}\underline{D} \tag{3}$$

where $\underline{r}, \underline{v}, \underline{\psi}$ are the position, velocity, and attitude error vectors, respectively. $\underline{f}, \underline{\delta g}, C_{LB}$ are the specific force, gravity error, and transition matrix from sensor (body) to navigation (LLLN –local north local level) frame. $\underline{B}, \underline{D}$ are the bias and drift error vectors, respectively.

The angular velocities are defined by:

$$\underline{w}_r = \underline{w}_{LE} \tag{4}$$

$$\underline{w}_v = \underline{w}_{LE} + 2\underline{w}_{EI} \tag{5}$$

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$$\underline{w}_{\psi} = \underline{w}_{LE} + \underline{w}_{EI}, \tag{6}$$

$\underline{w}_{LE}, \underline{w}_{EI}$ are the rotation vector from navigation frame to earth fixed frame and the rotation vector from the earth fixed frame to the inertial frame, respectively. See [5] for more details.

For medium (tactical) and low (automotive) quality inertial sensors the time of concern is relatively short, bias, drift, and attitude errors are relatively high, and, as it turns out, one can neglect the terms containing rotation vectors and the gravity error. Doing so, we obtain the following approximated navigation error model.

$$\dot{\underline{r}} = \underline{v} \tag{7}$$

$$\dot{\underline{v}} = \underline{f} \times \underline{\psi} + C_{LB} \underline{B} \tag{8}$$

$$\dot{\underline{\psi}} = -C_{LB} \underline{D} \tag{9}$$

Let us discuss the dynamic model of drift and bias. For proper GPS/INS integration, it is very important to model correctly variations in bias and drift. The standard approach is to define two components, random constant for constant drift or bias, and Markov process for drift (bias) variations. Let $D_k^{\ c}, B_k^{\ c}$ denote constant drift and bias at discrete time t_k, similarly $D_k^{\ m}, B_k^{\ m}$ denote discrete time Markov drift and bias. Then the discrete time system equations are written as follows:

$$r_{k+1} = r_k + v_k dt \tag{10}$$

$$v_{k+1} = v_k + F_k \psi_k + C_k B_k^{\ c} + C_k B_k^{\ m} + w_k^{\ v} \tag{11}$$

$$D_{k+1}^{\ c} = D_k^{\ c} \qquad D_{k+1}^{\ m} = \alpha^D D_k^{\ m} + w_k^{\ D} \tag{12}$$

$$B_{k+1}^{\ c} = B_k^{\ c} \qquad B_{k+1}^{\ m} = \alpha^B B_k^{\ m} + w_k^{\ B} \tag{13}$$

where F_k is the integral, from t_k till t_{k+1}, of the skew symmetric matrix corresponding to the specific force $\underline{f}$. Similarly, C_k is the integral of the transition matrix C_{LB}. Noise processes $w_k^{\ v}, w_k^{\ \psi}$ are due to accelerometer and gyro random walk, respectively. The time increment is denoted by dt.

In order to define precisely the filtering process, let us define the following new variables.

$$A_k^{\ c} = \frac{1}{dt}\left(F_k \psi_k + C_k B_k^{\ c}\right) \tag{14}$$

$$A_k^{\ m} = \frac{1}{dt} C_k^{\ m} B_k^{\ m} \tag{15}$$

$$J_k^{\ c} = \frac{-1}{dt^2} F_{k+1} C_k D_k^{\ c} \tag{16}$$

$$J_k^{\ m} = \frac{-1}{dt^2} F_{k+1} C_k D_k^{\ m} \tag{17}$$

For constant flight segments $\left(F_{k+2} = F_{k+1} = F_k\right)$ $\left(C_{k+1} = C_k\right)$ the new variables behave like constant acceleration, Markov acceleration, constant jerk and Markov jerk. Namely, it turns out that they satisfy the following equations:

$$r_{k+1} = r_k + v_k dt \tag{18}$$

$$v_{k+1} = v_k + A_k^{\ c} dt + A_k^{\ m} dt + w_k^{\ v} \tag{19}$$

$$A_{k+1}^{\ c} = A_k^{\ c} + J_k^{\ c} dt + J_k^{\ m} dt + w_k^{\ J} \tag{20}$$

$$A_{k+1}^{\ m} = \alpha^B A_k^{\ m} + w_k^{\ A} \tag{21}$$

$$J_{k+1}^{\ c} = J_k^{\ c} \tag{22}$$

$$J_{k+1}^{\ m} = \alpha^D J_k^{\ m} + w_k^{\ m} \tag{23}$$

where $w_k^{\ v}, w_k^{\ J}, w_k^{\ A}, w_k^{\ m}$ are corresponding components of process noise.

Observe that Equations (18-23) are trajectory invariant, moreover they describe time constant system. On the other hand they cover all sensor error model parameters. Equations (14-23) define the division of estimation algorithm into two processes. The filtering process is defined as an integration of Equations (18-23) with the GPS error model. The separation process is defined as a solution of Equations (14-17) with left hand sides assumed, as filtering process results, to be known. Note that the flight trajectory data, described by F_k and C_k appear only in separation problem set-up. All data about the sensor error model, process noise and Markov process parameters are included only in filtering problem description.

The above approach is abased on two assumptions:

- The navigation error model is described by Equations (7-9)
- The flight trajectory can be described as consists of constant segments connected by short-time turns.

In the following section, we describe numerical studies to justify the above assumptions.

3. PRELIMINARY NUMERICAL ANALYSIS

To carry out numerical studies we need to define precisely the error model and flight trajectories. We consider a parametric family of IMU's (Inertial Measurement Units) named Q1, Q10, Q100. The name convention is that Q1 is a prototype of a standard tactical quality IMU (for example, Honywell's HG1700, Litton's LN200, or Kearfott's T16B), Q100 is a prototype of automotive quality sensors. Q10 is a

theoretical model of a unit roughly 10 times worst than Q1. Table 1 describes their detailed error models. All Markov processes defined in Table 1 have 300 seconds correlation time.

Table 1 IMU's error model

Error	Q1	Q10	Q100
gyro constant drift	1 deg/h	10 deg/h	100 deg/h
gyro Markov drift	0.2 deg/h	2 deg/h	20 deg/h
gyro random walk deg/sqrt(h)	0.1	0.3	1.0
accelerometer constant bias	1mg	3 mg	10 mg
accelerometer Markov bias	0.2 mg	0.6 mg	2 mg
accelerometer random walk m/sec/sqrt(h)	0.03	0.1	0.3

GPS error model is given in Table 2. From many possible configurations, without effecting the final conclusions, SA off and position measurements are assumed

Table 2 GPS error model

Error	Variance	Correlation time
Horizontal position white	2.8 m	
Horizontal position Markov	10 m	20 sec.
Vertical position white	3.5 m.	
Vertical position Markov	13 m.	20 sec

The flight trajectories, analyzed in this paper, are level and consist of straight segments and turns. They have three parameters. The angle α is the change of the heading during turns, t_m is the time between two consecutive turns, and t_d is the turn duration.

Having defined all error models (IMU and GPS) it is straightforward to define the appropriate Kalman filter. This filter calculates the integrated system optimal performance. We will focus on the standard deviation of attitude errors, leveling ψ_x and heading ψ_z. Tables 3.1 and 3.2 present results for half an hour simulation with time between turns $t_m = 300$ seconds.

Table 3.1 Leveling error ψ_x for $\alpha = 15$

ψ_x mrad	Q1		Q10		Q100	
t_d	Prec.	Appr	Prec.	Appr	Prec.	Appr
1	0.58	0.70	1.71	2.08	5.59	6.83
5	0.58	0.70	1.71	2.09	5.60	6.85
10	0.58	0.70	1.71	2.09	5.66	6.91
20	0.58	0.71	1.73	2.11	5.85	7.13
50	0.59	0.72	1.83	2.23	6.96	8.23
100	0.64	0.77	2.07	2.47	8.64	9.49

Table 3.2 Heading error ψ_z for $\alpha = 15$,.

ψ_z mrad	Q1		Q10		Q100	
t_d	Prec	Appr	Prec	Appr	Prec	Appr
1	7.8	8.0	23.1	23.4	51	51
5	7.8	8.1	23.5	23.8	53	53
10	7.9	8.1	24.1	24.4	55	56
20	8.1	8.3	25.6	26.1	62	62
50	8.9	9.2	33.0	33.8	102	103
100	10.6	11.2	49.6	51.7	237	242

The column "Prec." corresponds to the simulation based on the precise error model (equations (1), (2), (3)) while the column "Appr." is related to approximated error model (equations (7), (8) (9)). From the comparison between corresponding columns, one can see that using the approximated strapdown error model introduces differences less than 20 %. This observation has twofold consequences:

- It justifies the approximated model
- It sets the accuracy level that should be expected in the subsequent analysis

It is interesting to consider the influence of turn duration t_d on the performance. It is clear from Tables 3.1 and 3.2 that for low values, 1-10 seconds, t_d does not influence the results. In other words, two different trajectories, with the same heading changes and the same time between turns, but with different turn duration time, yield the same performance. This is key observation for the idea to divide the estimation algorithms into two parts: time invariant filtering and trajectory dependent separation. Since for separation a flight trajectory with constant segments is required, the idea is to replace every turn with an equivalent one with very short duration time. The next step is to neglect measurements during those very short turns, namely to consider estimation, as in separation set-up, only during constant segments

In spite of the fact that for low t_d the above idea works very well, observe that for longer duration

times (> 20 seconds) there are some changes in performance. Therefore, it is very important to find maximal duration time that is still equivalent to a very short duration time, say $t_d = 1$ second. Figure 1 describes the normalized leveling error with respect to turn duration time t_d. The heading change is 45 degree. It is evident that even the normalized behavior depends on sensor quality

Table 4 compares two different numbers. The first one is the maximal duration time to preserve leveling accuracy within the limits of 10 percents. To explain the second number, consider the time-invariant model, described in equations (18-22) integrated with GPS, as defined in Table 2. The resulting Kalman filter achieves steady state, and then the integrated system is time–invariant as well. The second row in Table 4 describes the time constants for acceleration in this steady state system.

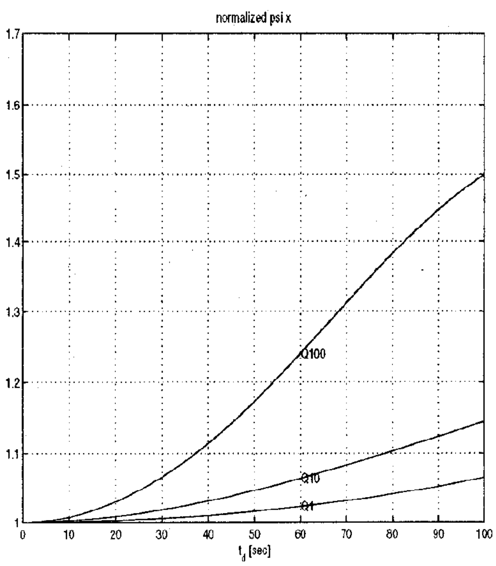

Figure 1 Normalized ψ_x

Table 4 Characteristic times comparison

	Q1	Q10	Q100
t_d for 10% accuracy	> 100	80	40
Time constant for acceleration in time-invariant system	121	75	36

The conclusion is apparent: as long as we deal with duration times less than time constants of the filtering process, we can reduce the duration time without changing significantly the performance. This conclusion has been further verified for a wide range of problem parameters. Now we can state the algorithm for approximate performance evaluation.

- Analyze the performance and time constants of filtering problem with respect to sensor error model.
- Divide the actual trajectory into segments no longer that the acceleration time constant as calculated for filtering problem. Preserve the heading change for every segment.
- Combine the results of filtering problem for different segments (separation process)

4 FILTERING AND SEPARATION

The solution of filtering problem is straightforward, it is carried out by Kalman filter which integrates dynamics described by Equations (18-23) with GPS error model specified in Table 2. The remaining separation problem is to solve Equations (14-17) assuming that:

- the flight trajectory consists of L constant segments,
- estimates and error covariance of accelerations and jerks $\left(A_k^{\ c}, A_k^{\ m}, J_k^{\ c}, J_k^{\ m}\right)$ are given.

Since flight trajectory segments are separated by time greater than filtering time constant, we can assume that acceleration and jerk estimates for different segments are statistically independent. This key observation provides the means to define a simple Kalman filter with attitude, biases, and drifts as states and acceleration and jerk as measurements. This approach yields accurate results, but further problem simplification is possible.

Consider again Equation (14), bias $B_k^{\ c}$ is constant but the attitude ψ_k, due to gyro drifts and gyro random walk, is time dependent. On the other hand, when acceleration estimate $\hat{A}_k^{\ c}$ was calculated, drift influences have been taken into account. Therefore, perhaps, one can consider a mean value version of leveling error. Namely, let us assume:

$$\hat{A}_k^{\ c} + v_k^{\ g} = \frac{1}{dt}\left(F_k \overline{\psi} + C_k B^c\right) \quad k = l_1, l_2, \ldots, l_L$$

where $\hat{A}_k^{\ c}, F_k, C_k, dt$ are given. The additional measurement noise $v_k^{\ g}$ is due to gyro random walk. The covariance matrix of the measurement errors R consists of σI, due to acceleration errors, and W due to gyro random walk. The problem has two unknown constant vectors $\overline{\psi}, B^c$. This is a linear constant parameter estimation and least square approach is the proper solution. Let us define

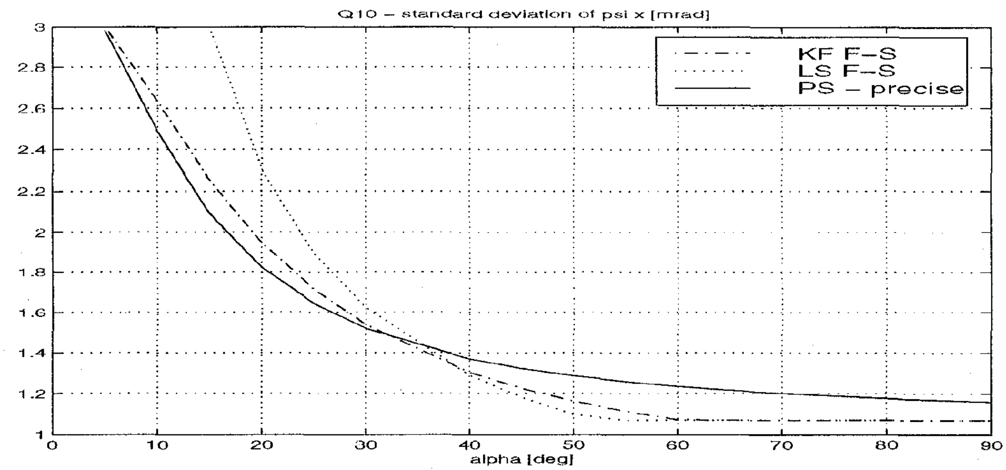

Figure 2 Leveling error with respect to heading change α - as calculated by different methods

$$T_m = \begin{bmatrix} F_{l_1} & C_{l_1} \\ F_{l_2} & C_{l_2} \\ \dots & \dots \\ F_{l_L} & C_{l_L} \end{bmatrix} \quad (24)$$

It is well known that the covariance matrix of the unknown parameters is given by:

$$P = \left(T_m^{\ T} R^{-1} T_m\right)^{-1} \quad (25).$$

Figure 2 describes the standard deviation of leveling error calculated by three means:

- LS F-S (least square filtering-separation) as described by Equations (24-25)
- KF F-S (Kalman filter filtering-separation)- Kalman filter solution of separation problem.
- PS (precise) – calculated by the complete simulation.

The first two results have been modified with respect to the following observations. The steady state leveling error can not be greater then the initial accelerometer bias error (multiplied by the gravity). The leveling error can not be less then the acceleration error (multiplied by the gravity) as calculated by the filtering problem

From Figure 2 one can conclude that the attitude accuracy, calculated by the filtering- separation scheme with a low dimensional Kalman filter, is very close to the one obtained by the entire system simulation. The above is true for all tested trajectories, those with large α, namely good estimability (see [3], [4] for an interesting discussion about observability of piece-wise constant systems) and those with small heading changes (poor estimability). Figure 2 depicts the results for Q10, the plots for Q1 and Q100 are very similar. The results of least square are very good for large heading changes, but are slightly worst for low estimability. Indeed, the least square is not able to take into account initial covariance, which, in this case, is an important factor.

5 CONCLUSION

It has been shown that the idea to divide the estimation process into filtering and separation provides simple and quite precise meanings for system performance evaluation. The most important feature of this approach is the independent treatment of the sensor error model and the flight trajectory.

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